Consideration of the Effects of $M_{2,min}$ on Slenderness Calculations for Non-Sway Column per CSA A23.3

Objective

Perform slenderness calculations for a square column that is part of a non-sway frame per CSA A23.3-04, CSA A23.3-14, and CSA A23.3-19 and compare results.

Codes

CSA A23.3-19, Design of Concrete Structures, Canadian Standards Association, 2019

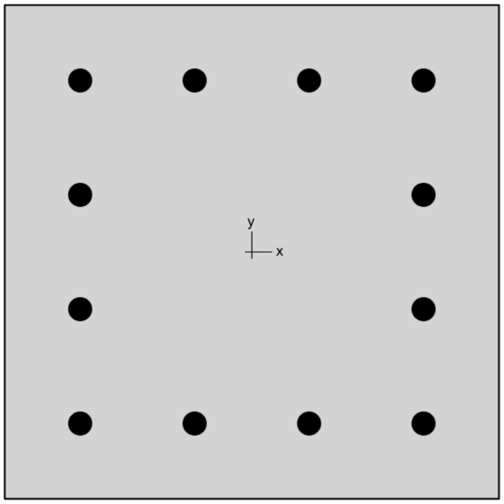
CSA A23.3-14, Design of Concrete Structures, Canadian Standards Association, 2014

CSA A23.3-04, Design of Concrete Structures, Canadian Standards Association, 2004

Contents

1. Non-sway Column Model – Geometry & Design Data

The cross-sectional view of the square concrete column below is generated from an analytical model using the spColumn Program.



Column dimensions: 400 x 400 mm

Reinforcement: 12 – 20M

Column unsupported height, l_u = 5.7 m

Effective length factor, k = 0.728

f_c' = 35 MPa

f_y = 400 MPa

$$r = \sqrt{\frac{I_g}{A_g}} = \sqrt{\frac{2.133E+09}{160000}} = 115.47 \text{ mm}$$

Plan view of a Non-sway Column

Load No.	Curvature	Axial Load, kN	Bending Moment, kN.m	
			Top	Bottom
1	Double	2775	60	1.0
2	Bottom moment is zero	2775	60	0.0
3	Single	2775	60	-1.0
4	Single, $M_2 > M_{2,min}$	2775	90	-1.0

Factored Axial Loads & First-order Moments

2. Slenderness Effects – Non-sway Frames

2.1 Slenderness Consideration Check

CSA A23.3-19, clause 10.15.2 states that in non-sway frames, slenderness effects may be ignored for compression members that satisfy the following equation:

$$\frac{k \times l_u}{r} \le \frac{25-10\left(\frac{M_1}{M_2}\right)}{\sqrt{\frac{P_f}{f_c' \times A_g}}}$$ **CSA A.23.3-19 (Eq. 10.16)**

where

per CSA A23.3-04 and CSA A23.3-14:

- M_1/M_2 is not taken less than -0.5.
- M_1/M_2 shall be taken positive if the member is bent in single curvature.

per CSA A23.3-19:

- M_1/M_2 is not taken less than -0.5.
- M_1/M_2 shall be taken positive if the member is bent in single curvature **and**
- **shall be taken as 1.0 if M_2 is less than $M_{2,min}$**

2.2 Moment Magnification Procedure

$$M_c = \frac{C_m M_2}{1-\frac{P_f}{\phi_m P_c}} \ge M_2$$ **CSA A23.3-19 (Eq. 10.17)**

where:

member resistance factor, $\phi_m = 0.75$ **CSA A23.3-19 (10.15.3.1)**

$$P_c = \frac{\pi^2 (EI)}{(kl_u)^2}$$ **CSA A23.3-04/14**

where:

(EI) may be taken as either of the following expressions

$$(EI) = \frac{0.2E_c I_g + E_s I_{st}}{1+\beta_d}$$ **CSA A23.3-04/14**

$$(EI) = \frac{0.4E_c I_g}{1+\beta_d}$$ **CSA A23.3-04/14**

CSA A23.3-19 introduced the effective flexural stiffness. $(EI)_{eff}$ expression as shown below:

$$P_c = \frac{\pi^2 (EI)_{eff}}{(kl_u)^2}$$ **CSA A23.3-19 (Eq. 10.18)**

where:

$(EI)_{eff}$ may be taken as the larger value from

$$(EI)_{eff} = \frac{0.2E_c I_g + E_s I_{st}}{1+\beta_d}$$ **CSA A23.3-19 (Eq. 10.19)**

$$(EI)_{eff} = \frac{0.4E_c I_g}{1+\beta_d}$$ **CSA A23.3-19 (Eq. 10.20)**

CSA A23.3-19, now, allows the use of the larger of the effective flexural stiffness, $(EI)_{eff}$. formulas for slender concrete columns. The first equation provides accurate representation of the reinforcement in the section and will, therefore, be used in this example and is also used by the solver in spColumn. Further comparison of the available options is provided in "Effective Flexural Stiffness for Critical Buckling Load of Concrete Columns" technical note.

M_2 in Equation 10.17 shall not be taken less than $M_{2,min}$ about each axis separately.

$$C_m = 0.6 + 0.4\frac{M_1}{M_2} \geq 0.4$$ **CSA A23.3-19 (Eq. 10.21)**

CSA A23.3-04, clause 10.15.3.1 stated that *"M_2 in Equation 10.16 shall not be taken as less than $P_f(15+0.03h)$ about each axis separately."*

CSA A23.3-14, clause 10.15.3.1 stated that *"M_2 in Equation 10.17 shall not be taken as less than $P_f(15+0.03h)$ about each axis separately* ***with the member bent in single curvature with C_m taken as 1.0.****"*

The CSA A23.3-14, clause 10.15.3.1 provides an incomplete guidance as it implies the M_2 shall not be taken less than the minimum moment, $P_f(15+0.03h)$ with members bent in single curvature only. This provision is revised entirely and clarified in CSA A23.3-19 as follows to consistently require C_m = 1.0 in all cases where $M_{2,min}$ exceeds M_2.

CSA A23.3-19, clause 10.15.3.1 states that *"M_2 in Equation 10.17 shall not be taken as less than $M_{2,min}$ about each axis separately.* ***If $M_{2,min}$ exceeds M_2, C_m shall be taken as equal to 1.0.****"*

3. Hand Solution

3.1 Load No 1: Member Bent in Double Curvature and $M_{2,min}$ exceeds M_2

Slenderness Consideration Check

M_1 = -1.0 kN.m

M_2 = 60.0 kN.m

M_1 / M_2 = -1.0 / 60.0 = -0.0167 (CSA A23.3-04/14)

Minimum moment, $M_{2,min}$: **<u>CSA A23.3-19 (10.15.3.1)</u>**

$$(M_2)_{min} = P_f(15+0.03h)$$

$$(M_2)_{min} = 2775\times(15+0.03\times400)/1000 = 74.92 \text{ kN.m} > M_2$$

Since M_2 is less than $M_{2,min}$, M_1/M_2 ratio shall be taken as 1.0 **(CSA A23.3-19)**

$$\frac{k\times l_u}{r} \le \frac{25-10\left(\frac{M_1}{M_2}\right)}{\sqrt{\frac{P_f}{f_c'\times A_g}}}$$

$$\frac{k\times l_u}{r} = \frac{0.728\times5700}{115.47} = 35.94$$

$$\frac{25-10\left(\frac{M_1}{M_2}\right)}{\sqrt{\frac{P_f}{f_c'\times A_g}}} = \frac{25-10(-0.0167)}{\sqrt{\frac{2,775,000}{35\times(400\times400)}}} = \frac{25.17}{0.704} = 35.75$$ (CSA A23.3-04/14)

$$\frac{k\times l_u}{r} = 35.94 > \frac{25-10\left(\frac{M_1}{M_2}\right)}{\sqrt{\frac{P_f}{f_c'\times A_g}}} = 35.75$$ ∴ Slenderness should be considered. (CSA A23.3-04/14)

$$\frac{25-10\left(\frac{M_1}{M_2}\right)}{\sqrt{\frac{P_f}{f_c'\times A_g}}} = \frac{25-10(1.0)}{\sqrt{\frac{2,775,000}{35\times(400\times400)}}} = \frac{15}{0.704} = 21.31$$ **(CSA A23.3-19)**

$$\frac{k\times l_u}{r} = 35.94 > \frac{25-10\left(\frac{M_1}{M_2}\right)}{\sqrt{\frac{P_f}{f_c'\times A_g}}} = 21.31$$ ∴ Slenderness should be considered. **(CSA A23.3-19)**

Moment Magnification Procedure

Determine the Critical Load (P_c)

$$P_c = \frac{\pi^2 (EI)_{eff}}{(kl_u)^2}$$ **CSA A23.3-19 (Eq. 10.18)**

where

$$(EI)_{eff} = \frac{0.2E_cI_g + E_sI_{st}}{1+\beta_d}$$ **CSA A23.3-19 (Eq. 10-19)**

With 12 – 20M reinforcement equally distributed on all sides and 400 mm x 400 mm column section

$$I_{st} = 0.176 \times \rho_t \times b \times h^3 \times \gamma^2$$

$$I_{st} = 0.176 \times \frac{12 \times 300}{400 \times 400} \times 400 \times 400^3 \times 0.695^2 = 4.897 \times 10^7 \text{ mm}^4$$

$$\beta_d = \frac{P_{f,sustained}}{P_f} = \frac{2775}{2775} = 1.0$$

$$(EI)_{eff} = \frac{0.2 \times (28.1647) \times (2.133 \times 10^9) + (200) \times (4.897 \times 10^7)}{1+1} = 1.09 \times 10^{10} \text{ kN.mm}^2$$

$$P_c = \frac{\pi^2 \times (1.09 \times 10^{10})}{(0.728 \times 5700)^2} = 6247 \text{ kN}$$

Determine the C_m value

$$C_m = 0.6 + 0.4\frac{M_1}{M_2} \geq 0.4$$ **CSA A23.3-19 (Eq. 10.21)**

Per CSA A23.3-04

$M_1 / M_2 = -1.0 / 60.0 = -0.0167$

$C_m = 0.6 + 0.4\frac{M_1}{M_2} = 0.6 + 0.4\frac{-1.0}{60.0} = 0.593 \geq 0.4$. Therefore, $C_m = 0.593$

Per CSA A23.3-14

$P_f(15 + 0.03h) = 2775 \times (15 + 0.03 \times 400)/1000 = 74.92 \text{ kN.m} > M_2$ $(M_2)_{min} = 74.92 \text{ kN.m} > M_2$

but the member is <u>NOT</u> bent in single curvature. Therefore, $M_1 / M_2 = -1.0 / 60.0 = -0.0167$

$C_m = 0.6 + 0.4\frac{M_1}{M_2} = 0.6 + 0.4\frac{-1.0}{60.0} = 0.593 \geq 0.4$. Therefore, $C_m = 0.593$

Per CSA A23.3-19

$(M_2)_{min} = 2775 \times (15 + 0.03 \times 400)/1000 = 74.92 \text{ kN.m} > M_2$. Therefore, $C_m = 1.0$

Calculate the Magnified Moment, M_c

$$M_c = \frac{C_m M_2}{1 - \frac{P_f}{\phi_m P_c}} \geq M_2$$ **CSA A23.3-19 (Eq. 10.17)**

Since $M_{2,min} > M_2$, take $M_2 = M_{2,\,min}$ in Eq. 10.17

$$M_c = \frac{0.593 \times 74.92}{1 - \frac{2775}{0.75 \times 6247}} = 109.03 \text{ kN.m}$$ (CSA A23.3-04/14)

$$M_c = \frac{1.0 \times 74.92}{1 - \frac{2775}{0.75 \times 6247}} = 183.73 \text{ kN.m}$$ **(CSA A23.3-19)**

3.2 Load No 2: Member Bottom Moment, M_1 = 0 and $M_{2,min}$ exceeds M_2

Slenderness Consideration Check

M_1 = 0.0 kN.m

M_2 = 60.0 kN.m

M_1 / M_2 = 0.0 / 60.0 = 0.0 (CSA A23.3-04/14)

Minimum moment, $M_{2,min}$: **CSA A23.3-19 (10.15.3.1)**

$$(M_2)_{min} = P_f(15 + 0.03h)$$

$$(M_2)_{min} = 2775 \times (15 + 0.03 \times 400)/1000 = 74.92 \text{ kN.m} > M_2$$

Since M_2 is less than $M_{2,min}$, M_1/M_2 ratio shall be taken as 1.0 **(CSA A23.3-19)**

$$\frac{k \times l_u}{r} \leq \frac{25 - 10\left(\frac{M_1}{M_2}\right)}{\sqrt{\frac{P_f}{f_c' \times A_g}}}$$

$$\frac{k \times l_u}{r} = \frac{0.728 \times 5700}{115.47} = 35.94$$

$$\frac{25 - 10\left(\frac{M_1}{M_2}\right)}{\sqrt{\frac{P_f}{f_c' \times A_g}}} = \frac{25 - 10(0.0)}{\sqrt{\frac{2,775,000}{35 \times (400 \times 400)}}} = \frac{25}{0.704} = 35.51$$ (CSA A23.3-04/14)

$$\frac{k \times l_u}{r} = 35.94 > \frac{25-10\left(\dfrac{M_1}{M_2}\right)}{\sqrt{\dfrac{P_f}{f_c' \times A_g}}} = 35.51$$ ∴ Slenderness should be considered. (CSA A23.3-04/14)

$$\frac{25-10\left(\dfrac{M_1}{M_2}\right)}{\sqrt{\dfrac{P_f}{f_c' \times A_g}}} = \frac{25-10(1.0)}{\sqrt{\dfrac{2,775,000}{35\times(400\times400)}}} = \frac{15}{0.704} = 21.31$$ **(CSA A23.3-19)**

$$\frac{k \times l_u}{r} = 35.94 > \frac{25-10\left(\dfrac{M_1}{M_2}\right)}{\sqrt{\dfrac{P_f}{f_c' \times A_g}}} = 21.31$$ ∴ Slenderness should be considered. **(CSA A23.3-19)**

Moment Magnification Procedure

Determine the Critical Load (P_c)

$$P_c = \frac{\pi^2 (EI)_{eff}}{(kl_u)^2}$$ **<u>CSA A23.3-19 (Eq. 10.18)</u>**

where

$$(EI)_{eff} = \frac{0.2E_cI_g + E_sI_{st}}{1+\beta_d}$$ **<u>CSA A23.3-19 (Eq. 10-19)</u>**

With 12 – 20M reinforcement equally distributed on all sides and 400 mm x 400 mm column section

$$I_{st} = 0.176 \times \rho_t \times b \times h^3 \times \gamma^2$$

$$I_{st} = 0.176 \times \frac{12\times300}{400\times400} \times 400 \times 400^3 \times 0.695^2 = 4.897\times10^7 \text{ mm}^4$$

$$\beta_d = \frac{P_{f,sustained}}{P_f} = \frac{2775}{2775} = 1.0$$

$$(EI)_{eff} = \frac{0.2\times(28.1647)\times(2.133\times10^9)+(200)\times(4.897\times10^7)}{1+1} = 1.09\times10^{10}\text{kN.mm}^2$$

$$P_c = \frac{\pi^2 \times \left(1.09 \times 10^{10}\right)}{\left(0.728 \times 5700\right)^2} = 6247 \text{ kN}$$

Determine the C_m value

$$C_m = 0.6 + 0.4\frac{M_1}{M_2} \geq 0.4$$ **CSA A23.3-19 (Eq. 10.21)**

Per CSA A23.3-04

$M_1 / M_2 = 0.0 / 60.0 = 0.0$

$C_m = 0.6 + 0.4\frac{M_1}{M_2} = 0.6 + 0.4\frac{0.0}{60.0} = 0.600 \geq 0.4$. Therefore, $C_m = 0.600$

Per CSA A23.3-14

$P_f(15 + 0.03h) = 2775 \times (15 + 0.03 \times 400)/1000 = 74.92 \text{ kN.m} > M_2$ $(M_2)_{\min} = 74.92 \text{ kN.m} > M_2$

but the member is NOT bent in single curvature. Therefore, $M_1 / M_2 = 0.0 / 60.0 = 0.0$

$C_m = 0.6 + 0.4\frac{M_1}{M_2} = 0.6 + 0.4\frac{0.0}{60.0} = 0.600 \geq 0.4$. Therefore, $C_m = 0.600$

Per CSA A23.3-19

$(M_2)_{\min} = 2775 \times (15 + 0.03 \times 400)/1000 = 74.92 \text{ kN.m} > M_2$. Therefore, $C_m = 1.0$

Calculate the Magnified Moment, M_c

$$M_c = \frac{C_m M_2}{1 - \dfrac{P_f}{\phi_m P_c}} \geq M_2$$ **CSA A23.3-19 (Eq. 10.17)**

Since $M_{2,\min} > M_2$, take $M_2 = M_{2,\min}$ in Eq. 10.17

$$M_c = \frac{0.600 \times 74.92}{1 - \dfrac{2775}{0.75 \times 6247}} = 110.25 \text{ kN.m}$$ (CSA A23.3-04/14)

$$M_c = \frac{1.0 \times 74.92}{1 - \dfrac{2775}{0.75 \times 6247}} = 183.73 \text{ kN.m}$$ **(CSA A23.3-19)**

3.3 Load No 3: Member bent in Single Curvature and $M_2 < M_{2,min}$

Slenderness Consideration Check

M_1 = 1.0 kN.m

M_2 = 60.0 kN.m

M_1 / M_2 = 1.0 / 60.0 = 0.0167 (CSA A23.3-04/14)

Minimum moment, $M_{2,min}$: **CSA A23.3-19 (10.15.3.1)**

$$(M_2)_{min} = P_f (15 + 0.03h)$$

$$(M_2)_{min} = 2775 \times (15 + 0.03 \times 400)/1000 = 74.92 \text{ kN.m} > M_2$$

Since M_2 is less than $M_{2,min}$, M_1/M_2 ratio shall be taken as 1.0 **(CSA A23.3-19)**

$$\frac{k \times l_u}{r} \leq \frac{25 - 10\left(\frac{M_1}{M_2}\right)}{\sqrt{\frac{P_f}{f_c' \times A_g}}}$$

$$\frac{k \times l_u}{r} = \frac{0.728 \times 5700}{115.47} = 35.94$$

$$\frac{25 - 10\left(\frac{M_1}{M_2}\right)}{\sqrt{\frac{P_f}{f_c' \times A_g}}} = \frac{25 - 10(0.0167)}{\sqrt{\frac{2,775,000}{35 \times (400 \times 400)}}} = \frac{24.83}{0.704} = 35.28$$ (CSA A23.3-04/14)

$$\frac{k \times l_u}{r} = 35.94 > \frac{25 - 10\left(\frac{M_1}{M_2}\right)}{\sqrt{\frac{P_f}{f_c' \times A_g}}} = 35.28$$ ∴ Slenderness should be considered. (CSA A23.3-04/14)

$$\frac{25 - 10\left(\frac{M_1}{M_2}\right)}{\sqrt{\frac{P_f}{f_c' \times A_g}}} = \frac{25 - 10(1.0)}{\sqrt{\frac{2,775,000}{35 \times (400 \times 400)}}} = \frac{15}{0.704} = 21.31$$ **(CSA A23.3-19)**

$$\frac{k \times l_u}{r} = 35.94 > \frac{25 - 10\left(\frac{M_1}{M_2}\right)}{\sqrt{\frac{P_f}{f_c' \times A_g}}} = 21.31$$ ∴ Slenderness should be considered. **(CSA A23.3-19)**

Moment Magnification Procedure

Determine the Critical Load (P_c)

$$P_c = \frac{\pi^2 (EI)_{eff}}{(kl_u)^2}$$ **CSA A23.3-19 (Eq. 10.18)**

where

$$(EI)_{eff} = \frac{0.2E_cI_g + E_sI_{st}}{1+\beta_d}$$ **CSA A23.3-19 (Eq. 10-19)**

With 12 – 20M reinforcement equally distributed on all sides and 400 mm x 400 mm column section

$$I_{st} = 0.176 \times \rho_t \times b \times h^3 \times \gamma^2$$

$$I_{st} = 0.176 \times \frac{12 \times 300}{400 \times 400} \times 400 \times 400^3 \times 0.695^2 = 4.897 \times 10^7 \text{ mm}^4$$

$$\beta_d = \frac{P_{f,sustained}}{P_f} = \frac{2775}{2775} = 1.0$$

$$(EI)_{eff} = \frac{0.2 \times (28.1647) \times (2.133 \times 10^9) + (200) \times (4.897 \times 10^7)}{1+1} = 1.09 \times 10^{10} \text{ kN.mm}^2$$

$$P_c = \frac{\pi^2 \times (1.09 \times 10^{10})}{(0.728 \times 5700)^2} = 6247 \text{ kN}$$

Determine the C_m value

$$C_m = 0.6 + 0.4\frac{M_1}{M_2} \geq 0.4$$ **CSA A23.3-19 (Eq. 10.21)**

Per CSA A23.3-04

$M_1 / M_2 = 1.0 / 60.0 = 0.0167$

$C_m = 0.6 + 0.4\frac{M_1}{M_2} = 0.6 + 0.4\frac{1.0}{60.0} = 0.607 \geq 0.4$. Therefore, $C_m = 0.607$

Per CSA A23.3-14

$M_1 / M_2 = 1.0 / 60.0 = 0.0167$

$C_m = 0.6 + 0.4\frac{M_1}{M_2} = 0.6 + 0.4\frac{1.0}{60.0} = 0.607 \geq 0.4$. Therefore, $C_m = 0.607$

$P_f(15 + 0.03h) = 2775 \times (15 + 0.03 \times 400)/1000 = 74.92 \text{ kN.m} > M_2 \quad (M_2)_{min} = 74.92 \text{ kN.m} > M_2$

The value of C_m may be taken as 1.0 per the incomplete guidance in CSA A23.3-14, clause 10.15.3.1. The reader is referred to the discussion in section 2.2. This clause is edited in its entirety in CSA A23.3-19 and removed the incomplete and vague guidance in CSA A23.3-14, clause

10.15.3.1. In CSA A23.3-19, it now clearly and consistently takes C_m equal to 1.0 in all non-sway columns where $M_{2,min}$ exceeds M_2.

Per CSA A23.3-19

$$\left(M_2\right)_{\min} = 2775 \times (15 + 0.03 \times 400) / 1000 = 74.92 \text{ kN.m} > M_2$$. Therefore, take C_m as equal to 1.0.

Calculate the Magnified Moment, M_c

$$M_c = \frac{C_m M_2}{1 - \dfrac{P_f}{\phi_m P_c}} \geq M_2$$ **CSA A23.3-19 (Eq. 10.17)**

Since $M_{2,min} > M_2$, take $M_2 = M_{2,\ min}$ in Eq. 10.17

$$M_c = \frac{0.607 \times 74.92}{1 - \dfrac{2775}{0.75 \times 6247}} = 111.48 \text{ kN.m}$$ (CSA A23.3-04/14)

$$M_c = \frac{1.0 \times 74.92}{1 - \dfrac{2775}{0.75 \times 6247}} = 183.75 \text{ kN.m}$$ **(CSA A23.3-19)**

3.4 Load No 4: Member bent in Single Curvature and $M_2 > M_{2,min}$

Slenderness Consideration Check

$M_1 = 1.0$ kN.m

$M_2 = 90.0$ kN.m

$M_1 / M_2 = 1.0 / 90.0 = 0.0111$ (CSA A23.3-04/14)

Minimum moment, $M_{2,min}$: **CSA A23.3-19 (10.15.3.1)**

$$\left(M_2\right)_{\min} = P_f (15 + 0.03h)$$

$$\left(M_2\right)_{\min} = 2775 \times (15 + 0.03 \times 400) / 1000 = 74.92 \text{ kN.m} < M_2$$

Since M_2 is NOT less than $M_{2,min}$, M_1/M_2 ratio shall be taken as 0.0111 **(CSA A23.3-19)**

$$\frac{k \times l_u}{r} \leq \frac{25 - 10\left(\dfrac{M_1}{M_2}\right)}{\sqrt{\dfrac{P_f}{f_c' \times A_g}}}$$

$$\frac{k \times l_u}{r} = \frac{0.728 \times 5700}{115.47} = 35.94$$

$$\frac{25-10\left(\frac{M_1}{M_2}\right)}{\sqrt{\frac{P_f}{f_c'\times A_g}}}=\frac{25-10(0.0111)}{\sqrt{\frac{2,775,000}{35\times(400\times400)}}}=\frac{24.89}{0.704}=35.36$$ (CSA A23.3-04/14/**19**)

$$\frac{k\times l_u}{r}=35.94>\frac{25-10\left(\frac{M_1}{M_2}\right)}{\sqrt{\frac{P_f}{f_c'\times A_g}}}=35.36$$ ∴ Slenderness should be considered.

Moment Magnification Procedure

Determine the Critical Load (P_c)

$$P_c=\frac{\pi^2(EI)_{eff}}{(kl_u)^2}$$ **CSA A23.3-19 (Eq. 10.18)**

where

$$(EI)_{eff}=\frac{0.2E_cI_g+E_sI_{st}}{1+\beta_d}$$ **CSA A23.3-19 (Eq. 10-19)**

With 12 – 20M reinforcement equally distributed on all sides and 400 mm x 400 mm column section

$$I_{st}=0.176\times\rho_t\times b\times h^3\times\gamma^2$$

$$I_{st}=0.176\times\frac{12\times300}{400\times400}\times400\times400^3\times0.695^2=4.897\times10^7\ mm^4$$

$$\beta_d=\frac{P_{f,sustained}}{P_f}=\frac{2775}{2775}=1.0$$

$$(EI)_{eff}=\frac{0.2\times(28.1647)\times(2.133\times10^9)+(200)\times(4.897\times10^7)}{1+1}=1.09\times10^{10}\,kN.mm^2$$

$$P_c=\frac{\pi^2\times(1.09\times10^{10})}{(0.728\times5700)^2}=6247\ kN$$

Determine the C_m value

$$C_m=0.6+0.4\frac{M_1}{M_2}\geq0.4$$ **CSA A23.3-19 (Eq. 10.21)**

Per CSA A23.3-04

$M_1 / M_2 = 1.0 / 90.0 = 0.0111$

$$C_m = 0.6 + 0.4\frac{M_1}{M_2} = 0.6 + 0.4\frac{1.0}{90.0} = 0.604 \geq 0.4$$. Therefore, $C_m = 0.604$

Per CSA A23.3-14

$$P_f(15 + 0.03h) = 2775 \times (15 + 0.03 \times 400)/1000 = 74.92 \text{ kN.m} < M_2$$

$$(M_2)_{\min} = 74.92 \text{ kN.m} < M_2$$

Therefore, $M_1 / M_2 = 1.0 / 90.0 = 0.0111$

$$C_m = 0.6 + 0.4\frac{M_1}{M_2} = 0.6 + 0.4\frac{1.0}{90.0} = 0.604 \geq 0.4$$. Therefore, $C_m = 0.604$

Per CSA A23.3-19

$$(M_2)_{\min} = 2775 \times (15 + 0.03 \times 400)/1000 = 74.92 \text{ kN.m} < M_2 .$$

Therefore, $M_1 / M_2 = 1.0 / 90.0 = 0.0111$

$$C_m = 0.6 + 0.4\frac{M_1}{M_2} = 0.6 + 0.4\frac{1.0}{90.0} = 0.604 \geq 0.4$$. Therefore, $C_m = 0.604$

Calculate the Magnified Moment, M_c

$$M_c = \frac{C_m M_2}{1 - \dfrac{P_f}{\phi_m P_c}} \geq M_2$$ **<u>CSA A23.3-19 (Eq. 10.17)</u>**

$M_2 = 90.0$ kN.m & $M_{2,min} < M_2$

$$M_c = \frac{0.604 \times 90.0}{1 - \dfrac{2775}{0.75 \times 6247}} = 133.29 \text{ kN.m}$$ (CSA A23.3-04/14/**19**)

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4. Summary and Comparison of Results

The comparison of the results per CSA A23.3-04, CSA A23.3-14 and CSA A23.3-19 are tabulated below:

Load No	1			2			3			4		
Unsupported Length of the Column, l_u (mm)	5700			5700			5700			5700		
$\frac{k \times l_u}{r}$	35.94			35.94			35.94			35.94		
$\sqrt{\frac{P_f}{f_c' \times A_g}}$	0.704			0.704			0.704			0.704		
M_1	-1.0			0.0			1.0			1.0		
M_2	60.0			60.0			60.0			90.0		
$M_{2,min}$	74.92			74.92			74.92			74.92		
Single Curvature (Yes / No)	No			No			Yes			Yes		
Applied Moment, M_2 is less than $M_{2,min}$ (Yes / No)	**Yes**			**Yes**			**Yes**			No		
Calculated $\left(\frac{M_1}{M_2}\right)$	-0.0167			0.000			0.0167			0.0111		
Calculated C_m	0.593			0.600			0.607			0.604		
CSA A23.3	**04**	**14**	**19**	**04**	**14**	**19**	**04**	**14**	**19**	**04**	**14**	**19**
$\left(\frac{M_1}{M_2}\right)$ to be used at formula below	-0.0167	-0.0167	**1.000**	0.000	0.000	**1.000**	0.0167	0.0167	**1.000**	0.0111	0.0111	0.0111
$25-10\left(\frac{M_1}{M_2}\right)$	25.17	25.17	**15.00**	25.00	25.00	**15.00**	24.83	24.83	**15.00**	24.89	24.89	24.89
$\frac{25-10\left(\frac{M_1}{M_2}\right)}{\sqrt{\frac{P_f}{f_c' \times A_g}}}$	35.75	35.75	**21.31**	35.52	35.52	**21.31**	35.28	35.28	**21.31**	35.36	35.36	35.36
Ignore Slenderness Effects IF $\frac{k \times l_u}{r} \le \frac{25-10\left(\frac{M_1}{M_2}\right)}{\sqrt{\frac{P_f}{f_c' \times A_g}}}$ (Yes / No)	No	No	No	No	No	No	No	No	No	No	No	No
Minimum "Unsupported Length" in order to Ignore Slenderness (mm)	5670	5670	**3380**	5633	5633	**3380**	5595	5595	**3380**	5608	5608	5608
Governing Moment, M_2 in M_c Equation	74.92	74.92	74.92	74.92	74.92	74.92	74.92	74.92	**74.92**	90.00	90.00	90.00
C_m value per Code	0.593	0.593	**1.000**	0.600	0.600	**1.000**	0.607	* 0.607	**1.000**	0.604	0.604	0.604
Moment Magnification Factor, δ	1.455	1.455	**2.452**	1.471	1.471	**2.452**	1.488	1.488	**2.452**	1.481	1.481	1.481
Magnified Factored Moment, Mc	109.03	109.03	**183.73**	110.25	110.25	**183.73**	111.48	111.48	**183.73**	133.29	133.29	133.29

***** Refer to the discussion in Section 2.2 for the determination of the C_m value.